Simulating acoustic waves in spotted stars

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ABSTRACT

Acoustic modes of oscillation are affected by stellar activity, however it is unclear how starspots contribute to these changes. Here we investigate the nonmagnetic effects of starspots on global modes with angular degree ℓ ≤ 2 in highly active stars, and characterize the spot seismic signature on synthetic light curves. We perform 3D time-domain simulations of linear acoustic waves to study their interaction with a model star spot. We model the spot as a 3D change in the sound speed stratification with respect to a convectively stable stellar background, built from solar Model S. We perform a parametric study by considering different depths and perturbation amplitudes. Exact numerical simulations allow the investigation of the wavefield-spot interaction beyond first order perturbation theory. The interaction of the axisymmetric modes with the star spot is strongly nonlinear. As mode frequency increases, the frequency shifts for radial modes exceed the value predicted by linear theory, while the shifts for the ℓ = 2, m = 0 modes are smaller than predicted by linear theory, with avoided-crossing-like patterns forming between the m = 0 and m = 1 mode frequencies. The nonlinear behavior increases with increasing spot amplitude and/or decreasing depth. Linear theory still reproduces the correct shifts for nonaxisymmetric modes. In the nonlinear regime the mode eigenfunctions are not pure spherical harmonics, but rather a mixture of different spherical harmonics. This mode mixing, together with the frequency changes, may lead to misidentification of the modes in the observed acoustic power spectra.

Key words. starspots – asteroseismology – stars: activity – Sun: helioseismology – Sun: activity

1. Introduction

1.1. Motivation: low degree stellar oscillations

Starspots are the main observed features of magnetic activity in stars and play a fundamental role in understanding stellar dynamos. They have been detected in many stars through the modulation of light curves (Mosser & Appourchaux 2009) and Doppler imaging (Strassmeier 2009), the latter suggesting polar and high- to mid-latitude concentrations of magnetic fields. García et al. (2010) unambiguously detected an activity cycle in a solar-like star with asteroseismology for the first time, using data collected by the Convection, Rotation and planetary Transits (CoRoT) mission (Auvergne et al. 2009). Other excellent data are available from the Kepler (Borucki et al. 2010) satellite and even more will come from the upcoming missions Transiting Exoplanet Survey Satellite (TESS) to be launched in 2017 (Ricker et al. 2014), and PLAnetary Transits and Oscillations of stars (PLATO), to be launched in 2024 (Rauer et al. 2014).

The surface distribution of stellar magnetic activity, in principle, can be measured with asteroseismology. Gizon (2002) investigated the challenge of spatially resolving surface magnetic activity in other stars. He concluded that it would be possible to discriminate between a polar cap distribution and equatorial band distributions of activity, but only for stars with an inclination angle higher than 40°, activity stronger than the Sun, and rotation strong enough that the individual m-components in the acoustic spectra could be resolved. In a later study, using an empirical model calibrated to the Sun, Chaplin et al. (2007) tested the ability to measure, with asteroseismology, the extension of a latitudinal activity band distribution in Sun-like stars, finding that the best prospects for detection are for stars with magnetic activity concentrated at low latitudes. More recently Santos et al. (2012) studied the indirect (nonmagnetic) effects on radial acoustic oscillations induced by the changes in the stellar stratification due to star spots. In the case of the Sun they found that the frequency changes are too small (by two orders of magnitude) to explain the observed shifts. They concluded that the indirect effects of starspots on the stellar stratification cannot be responsible for the observed changes in the acoustic oscillations, at least for a star with a solar-like level of activity.

1.2. Our approach: time-domain numerical simulations of waves and star spot

In this work we extend the investigation to starspots in stars with a higher level of activity: those stars likely have starspots with larger surface coverage, thus implying considerable changes in the internal stratification (e.g., in the sound speed). We also explore the possibility of identifying starspots through asteroseismic observations of highly spotted stars. We address two main questions: what are the changes in the mode frequencies and amplitudes in such stars? And, what is the starspots seismic signature in synthetic light curves?

We simplify the problem by focusing on the interaction between the acoustic wavefield and a single star spot with a polar geometry (such a configuration is compatible with Doppler observations of a young solar analogue with strong activity, Marsden et al. 2005). For the star spot we model only the indirect changes induced in the sound speed in the stellar interior. These
changes in principle are not small, therefore, unlike in Santos et al. (2012), a perturbative approach may not be appropriate, and we need to use direct numerical simulations accounting for the full 3D structure of the star. Direct 3D numerical simulations also provide synthetic observations, and therefore are a powerful tool to characterize how the observable quantities change depending on the 3D stellar background.

The remainder of this work is organized as follows: in the next section we introduce the GLocal Acoustic Spherical Simulator (GLASS) code, which extends the code developed by Hanasoge et al. (2006) to include treatment of the center of the star. GLASS simulates linear acoustic waves propagating through the full 3D stellar interior, in the time-domain. In Sects. 3 and 4 we describe the spot model employed, analyze the effect of the spot on acoustic modes, and discuss the changes to the eigenmodes in the nonlinear regime. Section 5 highlights the eigenmodes in the nonlinear regime. Section 5 highlights the observational consequences.

2. Numerical method

2.1. Time-domain pseudo-spectral simulations in spherical geometry

The GLASS code solves the 3D linearized hydrodynamic equations in a spherical domain containing the full star, from the center up to the stellar surface. The use of the linear approximation is justified since acoustic wave perturbations in the Sun and solar-like stars have much smaller amplitudes compared to vibration is justified since acoustic waves propagating through the full 3D stellar interior, in the time-domain. In Sects. 3 and 4 we describe the spot model employed, analyze the effect of the spot on acoustic modes, and discuss the changes to the eigenmodes in the nonlinear regime. Section 5 highlights the eigenmodes in the nonlinear regime. Section 5 highlights the observational consequences.

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where \( j \) identifies the \( j \)th grid point from the center. A safety offset parameter is also implemented (we found that 2 is the smallest value that removes the instability). Finally, a fourth-order tridiagonal compact low-pass filter (Lele 1992) is applied in the radial direction, to avoid spectral blocking (Hanasoge & Duvall 2007) and smooth the discontinuity in \( r \) caused by the spectral filter. The physical solutions are not affected by the filtering; radial modes (the only modes that propagate through the center of the star) are not filtered, and all the other modes are already evanescent at the points \( r < r_c \) (the lower turning point for an \( \ell = 1 \) mode at 5 mHz is 0.03 \( R_\odot \), i.e., the tenth gridpoint in the grid we employ, while \( j=1 = 2 \)). However the combined action of the filter with the time evolution scheme introduces damping in the wavefield, with an exponential dependence on frequency. This results in a lifetime of ~50 days for waves with frequency at 2 mHz and down to ~3 days at 5 mHz for the simulation setup we used, which employs a radial grid with \( n_{\text{rad}} = 800 \) grid points and a time step of 1 s.

2.3. Validation: comparison with ADIPLS normal modes

We validated GLASS through independent calculations of the theoretical modes of oscillations for our QS model. These modes are uniquely identified by three quantum numbers: the radial order \( n \), the angular degree \( \ell \), and the azimuthal order \( m \). With the ADIPLS software package (Christensen-Dalsgaard 2008), we computed the eigenfrequencies \( \omega_{0,\text{adipl}} \) (which are degenerate in \( m \) in the QS model) and eigenfunctions \( \xi_{0,\text{adipl}} \) of all the acoustic modes in the range \( 3 < n < 37 \) and up to \( \ell = 2 \). We then used the eigenfunctions given by ADIPLS to excite the initial wavefield displacement in GLASS. Test simulations were run in a temporal window of 10 days (solar time), using different initial conditions (by exciting either one or a few eigenmodes at the same time) and grid sizes. We also performed one simulation with all the modes excited and in a temporal window of 70 days, to use as reference for our study.

In all the tests performed, the extracted eigenfrequencies and eigenfunctions were compared with ADIPLS solutions and showed good agreement: the difference between ADIPLS and GLASS for the eigenfrequencies was below 0.16 \( \mu \)Hz (i.e., the frequency resolution of the simulations) and the maximum difference for the eigenfunctions was ~0.1%, after the damping was accounted for.

3. Frequency shifts: nonlinear dependence on perturbation amplitude

3.1. Spot model: perturbation in sound speed

We modeled the changes to the stratification caused by a starspot as a positive change \( \Delta c^2 \) in the squared sound speed, while the density and pressure were unchanged. This guaranteed the compatibility with the stabilization method used for the background model (for which a decrease in the sound speed would reintroduce convective instabilities). We defined the change in the sound speed as

\[
\Delta c^2(r, \theta) = \epsilon \sigma(r) f(r, r_c) g(\theta),
\]

with positive amplitude \( \epsilon \), a radial profile

\[
f(r, r_c) = \frac{1}{2} \left[ \cos \left( \frac{r - r_c}{\sigma} \right) + 1 \right] \exp \left[ - \frac{(r - r_c)^3}{2\sigma^2} \right]
\]

for \( |r - r_c| < \sigma \sigma \) and zero otherwise, and with a latitudinal profile

\[
g(\theta) = \frac{1}{2} \left[ \cos (\kappa \theta) + 1 \right]
\]

for \( \kappa \theta < \pi \) and zero otherwise, where \( \theta \) is the colatitude. The functions \( f \) and \( g \) have continuous derivatives everywhere and define a spot located at the north pole, with a Gaussian profile in radius multiplied by a raised cosine and a raised cosine profile in latitude. For the study the vertical and horizontal size of the spot were fixed by setting \( \sigma = 0.01 R_\odot \) and \( \kappa = 2.4 \), and we varied the depth \( R_c - r_c \) and the amplitude \( \epsilon \). We note that the choice of the coordinate system here is completely arbitrary, since we are studying the perturbation to a spherically symmetric (e.g., nonrotating) background, therefore the results of this work can be translated to a spot located at any point at the surface, via a rotation of the coordinate system.

3.2. Linear theory

We first discuss the effect of small perturbations, in order to determine for which parameter range the frequency shifts fall in the linear regime. We consider the normal mode solutions \( \xi_0(r, t) = \xi_{0,\text{norm}}(r) e^{-i\omega_{0,\text{norm}} t} \) of our reference QS model, solving the wave equation

\[
\rho_0 \omega_{0,\text{norm}}^2 \xi_0 = -\nabla(\nabla \cdot \xi_0) - \nabla(\nabla \cdot p_0) + \omega_0 \nabla \cdot (\rho_0 \xi_0) = \mathcal{H} \xi_0
\]

that can be derived from Eqs. (1)–(4). Since the reference background is spherically symmetric, the solutions are degenerate in \( m \). We consider now the perturbation \( \Delta \mathcal{H} \) to the wave operator \( \mathcal{H} \) due to a generic change in the sound speed (while retaining pressure and density constant), given by

\[
\Delta \mathcal{H} \xi_0 = -\nabla \left( \Delta c^2 \rho_0 \nabla \cdot \xi_0 \right);
\]

the expression for the linear frequency shift (see, e.g., Aerts et al. 2010) is

\[
\frac{\Delta \omega_{0,\text{lin}}}{\omega_{0,\text{lin}}} = \frac{\int \Delta c^2 (\nabla \cdot \xi_0)(\nabla \cdot \xi_0) \rho_0 dV}{2 \omega_{0,\text{lin}}^2 \int \| \xi_0 \|^2 \rho_0 dV},
\]

where integration is performed over the entire volume of the star. Both the surface integral and the perturbation to the mode inertia do not contribute to the change in \( \Delta \omega_{0,\text{lin}} \) in this case: the surface integral is zero because of the surface boundary conditions imposed for the pressure, and there is no contribution from the perturbation to the mode inertia, since the density of the background is unchanged. Finally, by exploiting the separable form of the polar spot profile (6), we may write the frequency shift in terms of a product of separate integrals for the horizontal and radial coordinates. It is worth noting that, unlike the case of splitting induced by rotation, the frequency shift depends only on \( |m| \) and therefore modes with the same \( |m| \) are still degenerate. This holds true also in the nonlinear case.

3.3. Numerical simulations

3.3.1. Initial conditions: \( \delta \)-function source

We performed our study in the parameter range \( 0.01 \leq \epsilon \leq 1 \) and \( 0.97 R_\odot \leq r_c \leq 1 \) \( R_\odot \). The simulations run for 70 days (solar time) to reach the desired accuracy of ~0.16 \( \mu \)Hz in the frequency domain. Wavefield displacement and velocity records were taken with a cadence of 60 s (solar time), mimicking the usual cadence of helio- and asteroseismic observations and in order to have a Nyquist frequency of ~8.3 mHz (above the maximum acoustic cutoff frequency of ~5.3 mHz). Starting from
3.3.2. Nonlinear frequency shifts

The simulated frequencies have been extracted by taking the SH transform coefficient $p_{lm}(r_0, t)$ of the wavefield pressure $p(r_0, \theta, \phi, t)$ at each timestep and at $r_0 = R_0 + 200$ km above the surface. This was followed by a Fourier transform in time to obtain the field $\tilde{p}_{lm}(r_0, \omega)$ and then the power spectrum $P_{lm}(\omega) = |\tilde{p}_{lm}(r_0, \omega)|^2$. Finally we divided each $(\ell, m)$ spectrum in chunks with size of 80 $\mu$Hz and centered on the peak closest to the original ADIPLS mode frequency. A least-squares Lorentzian fit was applied to extract frequency, amplitude, and half width at half maximum (HWHM) of each mode.

Figures 1 and 2 showed selected results obtained for the frequency shifts $\Delta \nu = \Delta \omega / 2\pi$ induced by the spot with respect to QS in the case of a polar spot located at $r_c = 0.98 R_0$ (in the following we will always show plots related to simulations performed for this value of $r_c$). Figure 1 shows the relative frequency shifts $\Delta \nu / \nu$ as a function of mode frequency $\nu = \omega / 2\pi$ for all the modes we excited, extracted from a simulation with a spot of amplitude $\epsilon = 0.4$. The linear frequency shifts calculated from Eq. (9) reproduce the behavior of nonaxisymmetric modes ($m \neq 0$). However for axisymmetric modes the frequency shifts deviate from the theoretical linear shifts, but with a different behavior depending on $\ell$, the most interesting being the case $\ell = 2$ (bottom panel). Here the relative frequency shifts for $m = 0$ modes are smaller than predicted, and even decrease with increasing mode frequency, while $\Delta \nu / \nu$ remains roughly constant for the $|m| = 1$ and 2 modes. As a consequence, at a frequency of $\sim 3000$ $\mu$Hz the $\Delta \nu / \nu$ for $m = 0$ modes cross the relative frequency shifts of the $m = 1$ modes. Above this crossing frequency the $m = 1$ modes have the largest frequency shifts. Results from the other simulations performed shown that this crossing frequency decreases either when the amplitude $\epsilon$ increases (as seen, e.g., in Fig. 2) or when $r_c$ is moved toward the surface, the latter indicating a stronger nonlinear response of the system to smaller surface changes than to bigger changes buried deeper in the convection zone.

Figure 2 shows the simulated frequency shifts $\Delta \nu$ against spot amplitudes $\epsilon$ and for all the modes with $\ell = 2, n = 12$. These modes in the reference QS model are degenerate with respect to the azimuthal order $n$, with a frequency $\nu_0 = 1970.50 \pm 0.16$ $\mu$Hz. Again linear theory successfully reproduces the frequency shift for nonaxisymmetric modes, shifts for $|m| = 1$ however start to deviate from linear behavior at $\epsilon \approx 0.8$. The frequency shifts for $m = 0$ modes on the contrary are nonlinear already at $\epsilon$ values of $\sim 0.2$, with $\Delta \nu$ values smaller than predicted by linear theory. A parabolic fit is able to model the shift, thus indicating that in this case a second order perturbative correction could recover the actual frequencies. The figure shows also the crossing frequency (horizontal dotted line) matching the mode frequency at $\epsilon \approx 0.92$, with a value of about 1985 $\mu$Hz (corresponding to a frequency shift $\Delta \nu \approx 14.5$ $\mu$Hz), decreased by $\sim 1000$ $\mu$Hz with respect to the case with $\epsilon = 0.4$, as already noted above.

4. Perturbations to the eigenfunctions

The polar spot introduces a nonspherically-symmetric perturbation to the background model. As a consequence the eigenfunctions depart from their original horizontal dependence and get mixed with other spherical harmonics. The extent of the mixing
depends on the amplitude of the perturbation. Because of the axisymmetric profile of the spot there is no mixing between spherical harmonics with different $m$. We investigate the effect of the mixing in the eigenfunctions by writing the radial displacement eigenfunction of a given mode for the model with the spot as

$$\tilde{e}_r^{\ell m}(r, \theta) = e^{-i\omega_{nlm}t} \sum_{\ell=0}^{\ell_{\text{max}}} d_{\ell}^{\ell m}(r) Y_{\ell}^{m}(\theta, \phi)e^{im\phi},$$

(11)

with the spherical harmonics $Y_{\ell}^{m}(\theta, \phi)$ given by

$$Y_{\ell}^{m}(\theta, \phi) = \sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}} P_{\ell}^{m}(\cos \theta)e^{im\phi},$$

where $\ell_{\text{max}}$ gives the spectral resolution of the SH transform and where we labeled with $\tilde{e}$ the new mixed eigenmode originally represented by a pure $Y_{\ell}^{m}$ in the QS model. Here $\tilde{e}_r^{\ell m}(r, \theta)$ is the meridional profile of $\tilde{e}_r^{\ell m}(r)$, expressed through a truncated series of Legendre polynomials (i.e., the latitudinal components of the $Y_{\ell}^{m}$)

$$\tilde{e}_r^{\ell m}(r, \theta) = \sum_{\ell=0}^{\ell_{\text{max}}} d_{\ell}^{\ell m}(r) \sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}} P_{\ell}^{m}(\cos \theta).$$

(12)

The amplitude coefficients $d_{\ell}^{\ell m}(r)$ with $\ell \neq \ell$ give the degree of mixing with $Y_{\ell}^{m}$ of the $n\ell m$ mode at the radial position $r$.

Figure 3 shows radial cuts taken at different colatitudes $\theta$ (i.e., at different angular distances from the center of the spot) of the scaled radial displacement eigenfunction $(r^2 \rho)^{1/2} \tilde{e}_r^{\ell}(r, \theta)$, for $\ell = 2$, $m = 0$, $n = 12$ (top panel) and for a radial mode with $n = 16$ (bottom panel), for a spot with $\epsilon = 0.4$ and $r_c = 0.98 R_\odot$. Each cut has been renormalized to its maximum. In the case of the quadrupolar mode the phase shift of $\tilde{e}_r$ inside the spot with respect to the QS solution increases from the center, and reaches its maximum at the edge of the spot (i.e., for $\theta = 60^\circ$). The shift then decreases, and $\tilde{e}_r$ smoothly matches the QS eigenfunction at $\theta = 90^\circ$. The phase shift has a different behavior in the case of the radial mode with $n = 16$, it is maximum at the center of the spot and then approaches zero at the antipodes ($\theta = 180^\circ$). This is in agreement with what was found by Santos et al. (2012), except that in our case the radial profile of the eigenfunction does not match the QS eigenfunction at the spot edge.

5. Synthetic power spectra for a polar spot

5.1. Synthetic light curves

Mode mixing affects observed light curves, as was already pointed out by Dziembowski & Goode (1996) and Cunha & Gough (2000) in the case of RoAp stars, since it changes the expected mode visibilities. The production of a realistic synthetic light curve is a nontrivial task, which requires modeling all the contributions to the emergent intensity at the photosphere in the observed wavelength range. Here we only model the contribution of the photospheric pressure perturbations to the intensity fluctuations $\tilde{I}$ induced by the acoustic wavefield. This in principle requires an explicit relation between the mode displacement and $I(\theta, \phi, t)$ at the stellar surface, which is rather complicated in general (see Toutain & Gouttebroze 1993).

For the sake of simplicity we assume that $I(\theta, \phi, t)$ is proportional to the Eulerian pressure perturbation $p(r, \theta, \phi, t)$ measured at $r_0 = R_\odot + 200$ km. We then express $I$ in the frequency domain as

$$I(\theta, \phi, \omega) \propto p(r_0, \theta, \phi, \omega) = \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} p_{\ell m}(r_0, \omega) Y_{\ell}^{m}(\theta, \phi),$$

(13)

where $p_{\ell m}$ are the SH coefficients of the pressure, obtained as described in Sect. 3.3 and containing all the contributions to the wavefield in the spectral component ($\ell, m$), including the mixing from other $\ell \neq \ell$ modes. We can explicitly quantify the mixing of a single mode by writing (see also Eq. (11))

$$\tilde{p}(r_0, \theta, \phi, \omega_{nlm}) = \sum_{\ell=0}^{\ell_{\text{max}}} \tilde{p}_{\ell m}(r_0, \omega_{nlm}) Y_{\ell}^{m}(\theta, \phi),$$

(14)

where $\omega_{nlm}$ is the mode frequency in the presence of the spot. The values of $p_{\ell m}(r_0, \omega_{nlm})$ with $\ell \neq \ell$ give the degree of the mixing.

Figure 4 shows the coefficients $|p_{\ell m}(r_0, \omega_{nlm})|$ of expansion (14) for the $n = 12, \ell = 2, m = 0$ mode for three different values of $\epsilon$. Mixing in the $\epsilon = 0.01$ case is almost absent. For
As an example we consider now four synthetic power spectra \( P(\omega) \) of a star with an inclination angle \( i \) of the polar axis with respect to the line of sight of 80°, for QS and three different amplitudes with \( \epsilon \) values of 0.01, 0.4, and 1, respectively. To create the spectra we followed the procedure outlined by Gizon & Solanki (2003): starting from the intensity fluctuations approximated by Eq. (13), we calculated the disk integrated intensity \( I(\omega) \) in the frequency domain, accounting for projection and limb-darkening effects (Pierce 2000). The resulting power spectra \( P(\omega) = [I(\omega)]^2 \) vs. \( \Delta \nu \) are shown in Fig. 5 for the \( \ell = 2, n = 12 \) multiplet. We note that because of the initial conditions we set (i.e., the choice made in Eq. (10) of using the same phase in exciting all of the modes), the \( |m| > 0 \) peaks in these spectra are twice as high as the peaks in a spectrum of acoustic oscillations resulting from stochastic excitation (that is the case for Sun-like stars). The \( \epsilon = 0.01 \) case falls in the linear regime: the visibility of the modes is the same as that of pure spherical harmonics (observed amplitude ratios correspond to Fig. 2 of Gizon & Solanki (2003), once the residual degeneracy in the \( m \neq 0 \) modes and the choice of the phase in the initial conditions (10) are taken into account) with very little mixing, as already shown in Fig. 4.

The same holds true in the case of \( \epsilon = 0.4 \) and 1 for nonaxisymmetric modes (the differences in shape and height between the peaks in the plots occur because the bin size is comparable to the HWHM of the peaks for these modes). The \( m = 0 \) mode, on the contrary, departs from the linear regime both in frequency shift (which for \( \epsilon = 1 \) becomes even smaller than that one for \( |m| = 1 \)) and observed amplitude, as a consequence of the mixing with other spherical harmonics.

6. Conclusion

Using 3D linear numerical simulations, we investigated the changes in global acoustic modes with \( \ell \leq 2 \) induced by a localized sound-speed perturbation with relative amplitude \( \epsilon \), mimicking the changes caused by a starspots with a polar cap configuration.

The interaction of the wavefield with a polar spot strongly affects the axisymmetric modes, which show a nonlinear behavior increasing with \( \epsilon \), for \( \epsilon \geq 0.2 \). The frequency shifts for radial modes exceed the shifts predicted by linear theory, while the shifts for the \( \ell = 2, m = 0 \) modes are smaller than predicted by linear theory and cross the \( |m| = 1 \) modes at a frequency that decreases with increasing \( \epsilon \). For modes with \( m \neq 0 \) linear theory successfully predicts the correct frequencies.

The nonlinear changes, with \( \epsilon \), in the mode frequencies and mode mixing (resulting from the distortion of eigenfunctions) will play a role in the correct identification of the modes. Strong mode mixing may also cause \( \ell > 2 \) modes to become visible in the observed spectrum (see, e.g., Dziembowski & Goode 1996).

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